Some Queueing Formulae

From:

1. www.cs.helsinki.fi/u/alanko/ska/materials/**QUEUEING**%20**SYSTEM**%20**FORMULAS**.doc
2. *Leonard Kleinrock,Queueing Systems- Volume1: Theory, Wiley-Interscience, 1975.*

Queueing System Notations

λ arrival rate ($1/\lambda$ mean interarrival time)
μ service rate ($1/\mu$ mean service time)

n number of jobs in the system (read: **queue length**)
n_q number of jobs waiting to receive service
N mean queue length (read: in the system)
N_q mean number of jobs waiting to receive service

r	response time	R	mean response time
w	waiting time	W	mean waiting time

M/M/1

Utilization, traffic intensity $\rho = \lambda/\mu$
Stability condition: $\rho < 1$

Number of jobs in the system

P{ no jobs in the system }	$p_0 = 1 - \rho$
P{ n jobs in the system }	$p_n = \rho^n(1-\rho)$
P{ number of jobs ≥ n}	ρ^n
mean queue length N	$E(n) = \rho/(1-\rho)$
variance of the queue length	$Var(n) = \rho/(1-\rho)^2$

Number of waiting jobs in the queue

P{number of waiting jobs is k}	$P\{n_q = k\} = 1-\rho^2$, k=0; $\rho^{k+1}(1-\rho)$, k>0
mean number of waiting jobs ($N_q = N - \rho$)	$E(n_q) = \rho^2/(1-\rho)$

Response time

mean response time R	$E(r) = (1/\mu) / (1-\rho) = 1 / (\mu - \lambda)$
variance of the response time	$Var(r) = E(r)^2$
P{ response time ≤ t }	$F(t) = 1 - e^{-(\mu-\lambda) t}$
0.90 fractile of the response time	$r_{.9} = 2.3 E(r)$

Waiting time

P{ waiting time ≤ t }	$F(t) = 1 - \rho e^{-(\mu-\lambda) t}$
mean waiting time W	$E(w) = \rho [(1/\mu) / (1-\rho)]$

M/M/m

Traffic intensity $\rho = \lambda/(m\mu)$

Stability condition: $\rho < 1$

Number of jobs in the system

P{ no jobs in the system } $p_0 = \{ (m\rho)^m / [m! (1-\rho)] + \sum^{m-1} (m\rho)^i / i! \}^{-1}$

P{ n jobs in the system } $p_n = [(m\rho)^n/n!] p_0, \quad n < m$

$[\rho^n m^m/m!] p_0, \quad n \geq m$

P{ number of jobs ≥ m} $p_q = (m\rho)^m / [m! (1-\rho)] p_0$

mean number of jobs N $E(n) = m\rho + \rho p_q / (1-\rho)$

Number of jobs in the queue

mean queue length ($N_q = N - m\rho$) $E(n_q) = \rho p_q / (1-\rho)$

Response time

mean response time R $E(r) = (1/\mu) \{1 + p_q / [m (1-\rho)]\}$

Waiting time

mean waiting time ($W= N_q/\lambda$) $E(w) = p_q / [m\mu (1-\rho)]$